Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **October/November 2018**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	*EITHER*: State or imply non-modular inequality $2^2(2x-a)^2 < (x+3a)^2$, or corresponding quadratic equation, or pair of linear equations $2(2x-a) = \pm(x+3a)$	B1	
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1	
	Obtain critical values $x = \frac{5}{3}a$ and $x = -\frac{1}{5}a$	A1	
	State final answer $-\frac{1}{5}a < x < \frac{5}{3}a$	A1	
	OR: Obtain critical value $x = \frac{5}{3}a$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	B1	
	Obtain critical value $x = -\frac{1}{5}a$ similarly	B2	
	State final answer $-\frac{1}{5}a < x < \frac{5}{3}a$ [Do not condone $\leqslant$ for $<$ in the final answer.]	B1	
		4	

Question	Answer	Marks	Guidance
2	Rearrange the equation in the form $ae^{2x} = b$ or $ae^{x} = be^{-x}$	M1	
	Obtain correct equation in either form with $a = 2$ and $b = 5$	A1	
	Use correct method to solve for x	M1	
	Obtain answer $x = 0.46$	A1	
		4	

Question	Answer	Marks	Guidance
3 (i)	Sketch a relevant graph, e.g. $y = x^3$	B1	
	Sketch a second relevant graph, e.g. $y = 3 - x$, and justify the given statement	B1	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
3(ii)	State or imply the equation $x = \left(2x^3 + 3\right) / \left(3x^2 + 1\right)$	B1	
	Rearrange this in the form $x^3 = 3 - x$, or commence work *vice versa*	B1	
		2	

Question	Answer	Marks	Guidance
3(iii)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 1.213	**A1**	
	Show sufficient iterations to 5 d.p. or more to justify 1.213 to 3 d.p., or show there is a sign change in the interval (1.2125, 1.2135)	**A1**	
		3	

Question	Answer	Marks	Guidance
4(i)	Obtain $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta + 2\cos 2\theta$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta} = -2\sin\theta - 2\sin 2\theta$	**B1**	
	Use $\mathrm{d}y/\mathrm{d}x = \mathrm{d}y/\mathrm{d}\theta \div \mathrm{d}x/\mathrm{d}\theta$	**M1**	
	Obtain correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form, e.g. $-\frac{2\sin\theta + 2\sin 2\theta}{2\cos\theta + 2\cos 2\theta}$	**A1**	
		3	
4(ii)	Equate denominator to zero and use any correct double angle formula	**M1***	
	Obtain correct 3-term quadratic in $\cos\theta$ in any form	**A1**	
	Solve for θ	**depM1***	
	Obtain $x = 3\sqrt{3}/2$ and $y = \frac{1}{2}$, or exact equivalents	**A1**	
		4	

Question	Answer	Marks	Guidance
5	Separate variables correctly and integrate at least one side	**B1**	
	Obtain term $\ln y$	**B1**	
	Obtain terms $2 \ln x - \frac{1}{2} x^2$	**B1+B1**	
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits	**M1**	
	Obtain correct solution in any form, e.g. $\ln y = 2\ln x - \frac{1}{2}x^2 + \frac{1}{2}$	**A1**	
	Rearrange as $y = x^2 \exp\left(\frac{1}{2} - \frac{1}{2}x^2\right)$, or equivalent	**A1**	
		7	

Question	Answer	Marks	Guidance
6(i)	Rearrange in the form $\sqrt{3}\sin x - \cos x = \sqrt{2}$	**B1**	
	State $R = 2$	**B1**	
	Use trig formulae to obtain α	**M1**	
	Obtain $\alpha = 30°$ with no errors seen	**A1**	
		4	

Question	Answer	Marks	Guidance
6(ii)	Evaluate $\sin^{-1}\left(\frac{\sqrt{2}}{R}\right)$	**B1ft**	
	Carry out a correct method to find a value of x in the given interval	**M1**	
	Obtain answer $x = 75°$	**A1**	
	Obtain a second answer e.g. $x = 165°$ and no others [Treat answers in radians as a misread. Ignore answers outside the given interval.]	**A1ft**	
		4	

Question	Answer	Marks	Guidance
7(i)	Use product rule	**M1***	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and obtain an equation in a single trig function	**depM1***	
	Obtain a correct equation, e.g. $3\tan^2 x = 2$	**A1**	
	Obtain answer $x = 0.685$	**A1**	
		5	

Question	Answer	Marks	Guidance
7(ii)	Use the given substitution and reach $a\int\left(u^2-u^4\right)\mathrm{d}u$	M1	
	Obtain correct integral with a = 5 and limits 0 and 1	A1	
	Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3-\frac{1}{5}u^5\right)$	M1	
	Obtain answer $\frac{2}{3}$	A1	
		4	

Question	Answer	Marks	Guidance
8(i)	*EITHER*: Multiply numerator and denominator by 1 + 2i, or equivalent, or equate to $x + iy$, obtain two equations in x and y and solve for x or for y	**M1**	
	Obtain quotient $-\frac{4}{5}+\frac{7}{5}$ i, or equivalent	**A1**	
	Use correct method to find either r or θ	**M1**	
	Obtain $r = 1.61$	**A1**	
	Obtain $\theta = 2.09$	**A1**	
	OR: Find modulus or argument of 2 + 3i or of 1 – 2i	**B1**	
	Use correct method to find r	**M1**	
	Obtain $r = 1.61$	**A1**	
	Use correct method to find θ	**M1**	
	Obtain $\theta = 2.09$	**A1**	
		5	
8(ii)	Show a circle with centre 3 – 2i	**B1**	
	Show a circle with radius 1	**B1ft**	Centre not at the origin
	Carry out a correct method for finding the least value of $\lvert z\rvert$	**M1**	
	Obtain answer $\sqrt{13}\ -1$	**A1**	
		4	

Question	Answer	Marks	Guidance
9(i)	State or imply the form $\dfrac{A}{2-x}+\dfrac{B}{3+2x}+\dfrac{C}{(3+2x)^2}$	**B1**	
	Use a correct method to find a constant	**M1**	
	Obtain one of $A=1$, $B=-1$, $C=3$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value [Mark the form $\dfrac{A}{2-x}+\dfrac{Dx+E}{(3+2x)^2}$, where $A=1$, $D=-2$ and $E=0$, B1M1A1A1A1 as above.]	**A1**	
		5	
9(ii)	Integrate and obtain terms $-\ln(2-x)-\dfrac{1}{2}\ln(3+2x)-\dfrac{3}{2(3+2x)}$	**B3ft**	The f.t is on A, B, C; or on A, D, E.
	Substitute correctly in an integral with terms $a\ln(2-x)$, $b\ln(3+2x)$ and $c/(3+2x)$ where $abc\neq 0$	**M1**	
	Obtain the given answer after full and correct working [Correct integration of the A, D, E form gives an extra constant term if integration by parts is used for the second partial fraction.]	**A1**	
		5	

Question	Answer	Marks	Guidance
10(i)	*EITHER*: Expand scalar product of a normal to m and a direction vector of l	**M1**	
	Verify scalar product is zero	**A1**	
	Verify that one point of l does not lie in the plane	**A1**	
	OR: Substitute coordinates of a general point of l in the equation of the plane m	**M1**	
	Obtain correct equation in λ in any form	**A1**	
	Verify that the equation is not satisfied for any value of λ	**A1**	
		3	
10(ii)	Use correct method to evaluate a scalar product of normal vectors to m and n	**M1**	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer 74.5° or 1.30 radians	**A1**	
		3	
10(iii)	*EITHER*: Using the components of a general point P of l form an equation in λ by equating the perpendicular distance from n to 2	**M1**	
	OR: Take a point Q on l, e.g. (5, 3, 3) and form an equation in λ by equating the length of the projection of QP onto a normal to plane n to 2	**M1**	
	Obtain a correct modular or non-modular equation in any form	**A1**	
	Solve for λ and obtain a position vector for P, e.g. $7\mathbf{i} + 5\mathbf{j} + 7\mathbf{j}$ from $\lambda = 3$	**A1**	
	Obtain position vector of the second point, e.g. $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ from $\lambda = -1$	**A1**	
		4	